C 40602 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS–UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Maximum marks 25.

1. State sequential criterian for continuity.

2. Show that the sine function is continuous on $\mathbb{R}$.

3. Define Lipchitz function. If $f : A \to \mathbb{R}$ is a Lipschitz function then show that f is uniformly continuous on A.

4. Define tagged partition.

5. Show that every constant function on $[a,b]$ is in $\mathbb{R}[a,b]$.

6. State Cauchy's criterion for Riemann integrability.

7. Let F, G be differentiable on $[a,b]$ and let $f = F'$ and $g = G'$ belong to $\mathbb{R}[a,b]$, then show that

$$\int_a^b f\,G = [FG]_a^b - \int_a^b F\,g.$$

8. Show that $\lim\left(\frac{x}{n}\right) = 0$ for $x \in \mathbb{R}$.

9. Define uniform convergence of a sequence of functions.

10. State bounded convergence theorem.

11. State Weirstrass M-test for the uniform convergence of series of functions.

12. Evaluate $\int_1^\infty \frac{dx}{x^2+1}$.

Turn over

13. Find the principal value of $\int_{-2}^{3} \frac{dx}{(x-1)^3}$.

14. Discuss the absolute convergence of $\int_{0}^{\infty} \frac{\sin x}{n+1}\, dx$ for $n\pi \leq x \leq (n+1)\,\pi,\, n = 0,1,2,\ldots$

15. If $\int_{0}^{b} \frac{dx}{1+ax} = \frac{1}{a}\ln(1+ab)$. Evaluate $\int_{0}^{b} \frac{xdx}{(1+ax)^2}$.

Section B

Questions 16–23, *answer any number of questions.*
Each question carries 5 *marks.*
Maximum marks 35.

16. State and prove Boundedness theorem for continuous function.

17. Show that $f(x) = \frac{1}{1+x^2}, x \in \mathbb{R}$ is uniformly continuous in $\mathbb{R}$.

18. State and prove Squeeze theorem for Riemann integrable functions.

19. If $f \in \mathbb{R}[a,b]$ and f is continuous at a point $c \in [a,b]$. Then show that the indefinite integral

    $\mathrm{F}(z) = \int_{a}^{z} f$ for $z \in [a,b]$ is differentiable at c and $\mathrm{F}'(c) = f(c)$.

20. Show that a sequence (f_n) of bounded functions on $\mathrm{A} \subset \mathbb{R}$ converges uniformly on A to f iff $\| f_n - f \| n \to 0$.

21. Discuss the convergence of $f_n(x) = \frac{x^n}{n+x^n}, x \geq 0$. Is the convergence uniform on $[0,\infty]$.

22. Evaluate $\int_{-1}^{1} \frac{dx}{x^2-1}$.

23. Show that ~~r~~ $q \in \mathbb{R}, \int_{1}^{\infty} x^q\, e^{-x}\, dx$ converges.

Section C

Questions 24–27, *answer any* **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Maximum Minimum Theorem.

25. State and prove Cauchy's criterion of Riemann integrability.

26. Let (f_n) be a sequence of functions in $\mathbb{R}[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f. Then show that $f \in \mathbb{R}[a,b]$.

27. Show that $\int_0^{\infty} \frac{\sin x}{x} dx$ exists and converges to a finite real value and that this integral does not converge absolutely.

(2 × 10 = 20 marks)