C 41232 (Pages : 3) Name..................................

Reg. No...............................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MTS 4C 04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{-x}{y}, y(4) = -3$.

2. Solve $(x^2 - 9)\dfrac{dy}{dx} + xy = 0$.

3. Find the value of k so that the differential equation $(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0$ is exact.

4. Verify that the functions e^{-3x}, e^{4x} form a fundamental set of solutions of the differential equation $y'' - y' - 12y = 0$ on $(-\infty, \infty)$.

5. The function $y_1 - e^x$ is a solution of $y'' - y — 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution y_2.

6. Find the general solution of $y''' - 4y'' - 5y' = 0$.

7. Solve the initial value problem $y'' + 4y' + 5y = 35e^{-4x}, y(0) = -3, y'(0) = 1$.

8. Find $\mathscr{L}(f(t))$, where $f(t) = \sin 2t \cos 2t$.

9. Evaluate $\mathscr{L}^{-1}\left(\dfrac{s}{(s-2)(s-3)(s-6)}\right)$.

Turn over

10 Write $f(t)=\begin{cases}2, & 0 \le t < 3\\ -2, & t \ge 3\end{cases}$ in terms of unit step functions and find $\mathcal{L}\,(f(t))$

11. Show that the functions $f_1(x)=e^x, f_2(x)=xe^{-x}-e^{-x}$ are orthogonal on [0, 2],

12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2}+6\frac{\partial^2 u}{\partial x \partial y}+9\frac{\partial^2 u}{\partial y^2}$ is parabolic.

Section B

Answer any number of questions.
Each question carries 5 marks. Ceiling is 30.

13. Solve $x\frac{dy}{dx}+y=x^2y^2$.

14. Solve $\frac{dy}{dx}=(x+y+1)^2$ by using an appropriate substitution.

15. $y''' + y'' = e^x \cos x$.

16. Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$.

17. Solve $y' + 6y = e^{4t}, y(0) = 2$ using the Laplace transform.

18. Evaluate $\mathcal{L}^{-1}\left(\frac{1}{\left(s^2+k^2\right)^2}\right)$.

19. Expand $f(x) = x, -2 < x < 2$ in a Fourier series.

Section C

Answer any **one** *question.*

The question carries 10 *marks.*

20. Solve the initial value problem $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 17$ using Laplace transform.

21. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$ in a Fourier series.