C 80917 *CS* **(Pages : 4)** **Name..............................**

Reg. No..............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write Euler-Cauchy equation.
2. State the first shifting theorem for Laplace transforms.
3. Define odd function. Give an example.
4. What do you mean by a periodic function ? Give an example.
5. Find $L\left(t+e^{t}\right)$.
6. Find $L^{-1}\left(\frac{s}{s^{2}-a^{2}}\right)$.
7. If $L\left(f\left(t\right)\right)$ and $f'\left(t\right)$ exists, find $L\left(f'\left(t\right)\right)$.
8. Define Half range Fourier sine series.
9. Write one dimensional wave equation.
10. Write the characteristic equation of the equation $y''+10y'+29y=0$.
11. Write the error estimate the Trapezoidal rule.
12. Find the Wronskian of y_1, y_2 where $y_1=\cos x, y_2=\sin x$.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $y'' + y = 0,\ y(0) = 3,\ y(\pi) = -3.$

14. Find a basis of solutions for $x^2 y'' - xy' + y = 0.$

15. Solve $\left(D^2 + w^2\right) y = 0.$

16. Solve $x^2 y'' - 2.5\, xy' - 2y = 0.$

17. Find a particular solution of $y'' - 3y' - 4y = -8e^t \cos 2t.$

18. Show that Laplace transform is a linear operator.

19. Find the Laplace transform of $\sinh at$.

20. Find $L^{-1}\left(\dfrac{1}{(s-1)^4}\right).$

21. Find $L\left(\dfrac{1-e^t}{t}\right).$

22. Find the Fourier series of $f(x) = x - x^2, -\pi < x < \pi, f(x + 2\pi) = f(x).$

23. A town wants to drain and fill a small polluted swamp. The swamp averages 5 feet deep. About how many cubic yards of dirt will it take to fill the area after the swamp is drained.

24. Show that the function $y = e^x \cos y$ is a solution of the two dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve the non-homogeneous equation :

$y'' - y' - 2y = 10 \cos x.$

26. Solve the differential equation :

$$\left(D^2 + 4D + 4\right) y = \frac{e^{-2x}}{x^2}.$$

27. Find the inverse transform of $\frac{1}{s\,(s+1)\,(s+2)}$.

28. Find $L\,(t \sin at)$.

29. Solve :

$$y\,(t) = t^3 + \int_0^1 \sin\,(t-u)\,y\,(u)\,du.$$

30. Find the Fourier series for $f\,(x) = |\,x\,|$ is $[-\pi, \pi]$ with $f\,(x + 2\pi) = f\,(x)$.

31. Find the approximate solution to $y' = 1 + y^2, y\,(0) = 0$.

32. Compare the values of $\int_0^1 x\,dx$ obtained by using Trapezoidal and Simpson's rule.

33. Given $y' = -y, y\,(0) = 1$. Find the value of y' at x, $x = (0.01)\,(0.01)\,(0.04)$ by improved Euler method.

(6 × 5 = 30 marks)

Part D

*Answer any **two** questions.*
Each question carries 10 marks.

34. Solve : $x^2 y'' - zxy' + 2y = \left(3x^2 - 6x + 6\right)e^x$

$y(1) = 2 + 3e \qquad y'(1) = 30.$

35. Find the inverse transform of $\frac{1}{s^2}\left(\frac{s+1}{s^2+9}\right)$.

36. Find the Fourier series of $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$.

Hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots = \frac{\pi^2}{12}$.

(2 × 10 = 20 marks)