D 71693   (Pages : 3)   Name...................................

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THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours   Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Write the general form of first order ODE.

2. What do you mean by exact differential equation ?

3. Define dot product of two vectors.

4. State Cayley Hamilton theorem.

5. When will you say two matrices are equivalent ?

6. Define curl of a function.

7. Find the resultant of the vectors $p = [2, 4, -5], q = [1, -6, 9]$.

8. Define characteristic polynomial of a matrix.

9. What is the order of the differential equation $y \left(\dfrac{dy}{dx} \right)^3 + 8x = 0.$

10. Find the directional derivative of $f = x^2 + y^2$ at (1, 1) in the direction of $2i - 4j$.

11. Write the general form of Bernoulli differential equation.

12. State Gauss's divergence theorem.

                                          (12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Verify that $\dfrac{c}{x}$ is a solution of the differential equation $xy' = -y, c$ is a constant and $x \neq 0$.

14. Find the curve through the point (1, 1) in the xy-plane having at each of its points the slope $-\dfrac{y}{x}$.

                                          Turn over

15. Solve $2xyy' = y^2 - x^2$.

16. Let $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find the distance between u and v.

17. Find the projection of $a = [1, -3, 4]$ in the direction of $b = [3, 4, 7]$.

18. Find the unit tangent vector T to the curve $C = F(t) = \left(t^2, 3t - 2, t^3, t^2 + 5\right)$ in R^4 when $t = 2$.

19. Find the component of $(1, 1, 3)$ in the direction of $(0, 0, 5)$.

20. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x) = 2x^3 - 4x + 5$. Find $f(A)$.

21. Show that $\operatorname{curl}(u + v) = \operatorname{curl} u + \operatorname{curl} v$.

22. Show that every elementary matrix E is invertible, and its inverse is an elementary matrix.

23. Show that $\int_{(0,\pi)}^{\left(3, \frac{\pi}{2}\right)} e^x (\cos y \, dx - \sin y \, dy)$ is path independent.

24. Find the length of the curve $r(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any six questions.
Each question carries 5 marks.

25. Find all the curves in xy-plane whose tangents pass through the point (a, b).

26. Solve $\cos(x + y) dx + \left(3y^2 + 2y + \cos(x + y)\right) dy = 0$.

27. Find an integrating factor and solve the initial value problem

$$\left(e^{x+y} + ye^y\right) dx + \left(xe^y - 1\right) dy = 0, y(0) = -1.$$

28. Find the straight line L_1 through the point P : (1, 3) in the xy-plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

29. Find the volume of the tetrahedron with vertices (0, 0, 0), (1, 2, 0), (3, –3, 0), (1, 1, 5).

30. Show that the integral $\int_c F.dr = \int_c 2x \, dx + 2y \, dy + 4z \, dz$ is path independent in any domain in space and find its value in the integration from A : (0, 0, 0) to B : (2, 2, 2).

31. Describe the region and evaluate $\int_0^1 \int_{x^2}^x (1 - 2xy) \, dy \, dx$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1 + \cos\theta)$.

33. Verify Greens theorem in the plane for $F = \left[-y^3, x^3\right]$ and the region is the circle $x^2 + y^2 = 25$.

$(6 \times 5 = 30 \text{ marks})$

Part D

*Answer any **two** questions.*
Each question carries 10 marks.

34. Let $A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}$.

    (a) Find all eigen values of A.

    (b) Find a maximal set S of non-zero orthogonal eigenvectors of A.

    (c) Find an orthogonal matrix P such that $D = P^{-1} AP$ is diagonal.

35. Solve :

    (a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0, y(2) = \sqrt{\dfrac{\pi}{2}}$.

    (b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate the integral by divergence theorem $F = \left[z - y, y^3, 2z^3\right]$, S the surface of $y^2 + z^2 \leq 4, -3 \leq x \leq 3$.

$(2 \times 10 = 20 \text{ marks})$