C 40605 **(Pages : 4)** **Name.......................................**

Reg. No...................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTI VARIABLE

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Questions 1–15, *Answer any number of questions.*
Each question carries 2 *marks.*
Maximum marks 25.

1. Find the domain and range of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$.

2. Show that $\lim\limits_{(x, y) \to (0, 0)} \dfrac{xy^2}{x^2 + y^4}$ does not exist.

3. Find f_x and f_y if $f(x, y) = y^x$.

4. Find the Directional Derivative of $f(x, y) = 4 - x^2 - \dfrac{y^2}{4}$ at $(1, 2)$ in the direction of $\vec{u} = \cos\dfrac{\pi}{3}\,\hat{i} + \sin\dfrac{\pi}{3}\,\hat{j}$.

5. Find the gradient of $f(x, y) = y \ln x + xy^2$ at the point $(1, 2)$.

6. Find the relative extrema of $f(x, y) = 1 - \left(x^2 + y^2\right)^{1/3}$.

Turn over

7. Evaluate $\iint_{R} 2x - y \, dA$ where R is the region bounded by the parabola $x = y^2$ and the straight line $x - y = 2$.

8. Find the surface area of the portion of the plane $z = 2 - x - y$ that lies above the circle $x^2 + y^2 \le 1$ in the first quadrant.

9. Find a transformation T from a region S to the region R bounded by the lines $x - 2y = 0, x - 2y = -4, x + y = 4$ and $x + y = 1$ such that S is a rectangular region.

10. Sketch the region of integration and reverse the order of integration in $\int_{1}^{e} \int_{0}^{\ln x} f(x, y)\, dy\, dx$.

11. Check whether the vector field $\vec{F}(x, y, z) = 2xy\,\hat{i} + \left(x^2 + z^2\right)\hat{j} + 2yz\,\hat{k}$ is irrotational.

12. Find div $\left(\text{curl } \vec{F}\right)$ if $\vec{F}(x, y, z) = xyz\,\hat{i} + y\hat{j} + z\hat{k}$.

13. Use Green's theorem to evaluate $\oint_{C} \left(x^2 y + x^3\right) dx + 2xy\, dy$ where C if the boundary of the region bounded by $y = x$ and $y = x^2$.

14. State Gauss Divergence theorem.

15. Evaluate the surface integral $\iint_{S} x + z \, dS$ where S is the first octant portion of the cylinder $y^2 + z^2 = 9$ between $x = 0$ and $x = 4$.

Section B

Questions 16–23, *Answer any number of questions.*
Each question carries 5 *marks.*
Maximum marks 35.

16. Show that the function $z = \sqrt{x^2 + y^2} \tan^{-1} \frac{y}{x}$ satisfies $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$.

17. Let $w = x^2 y - xy^3$ where $x = \cos t$ and $y = e^t$. Find $\frac{dw}{dt}$ at $t = 0$.

18. Find the points on the sphere $x^2 + y^2 + z^2 = 14$ at which the tangent plane is parallel to the plane $x + 2y + 3z = 12$.

19. Find the relative extrema of $f(x, y) = 2x^2 + y^2 + 8x - 6y + 20$.

20. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, y = 0, z = 0$ and $2x + y = 2$.

21. Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} \, dx \, dy$ by changing the order of integration.

22. Find an equation of the tangent plane to the paraboloid

    $\vec{r}(u, v) = u\hat{i} + v\hat{j} + \left(u^2 + v^2\right)\hat{k}$ at the point $(1, 2, 5)$.

23. Find the surface integral $\iint_S y^2 + 2yz \, d\mathrm{S}$ where S is the first octant portion of the plane $2x + y + 2z = 6$.

Turn over

Section C

Questions 24–27, Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Find the second order partial derivatives of $w = \cos(2u - v) + \sin(2u + v)$.

(b) Let $z = f(x, y) = 2x^2 - xy$. Find Δz and use the result to find the change in z if (x, y) changes from $(1, 1)$ to $(0.98, 1.03)$.

25. Find the absolute maximum and absolute minimum values of the function $f(x, y) = 2x^2 + y^2 - 4x - 2y + 3$ on the rectangle $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.

26. Evaluate $\iiint_T \sqrt{x^2 + z^2}\, dv$ where T is the region bounded by the cylinder $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

27. verify Divergence Theorem for $\vec{F}(x, y, z) = 2z\hat{i} + x\hat{j} + y^2\hat{k}$ and T is the solid region bounded between the paraboloid $z = 4 - x^2 - y^2$ and the xy plane.

(2 × 10 = 20 marks)