C 20648 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find the general solution of the differential equation $\frac{dy}{dt} = -ay + b$ where a,b are positive real numbers.

2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.

3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.

4. Find the solution of the differential equation :

   $$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, y(0) = -1.$$

5. Find the Wronskian of the functions $\cos^2\theta, 1 + \cos(2\theta)$.

6. Find the general solution of the differential equation $y'' + 2y' + 2y = 0$.

7. Let $y = \phi(x)$ be a solution of the initial value problem :

   $$\left(1 + x^2\right)y'' + 2xy' + 4x^2y = 0,\ y(0) = 0, y'(0) = 1.$$

   Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.

Turn over

9. Find the Laplace transform of' the function sin (at).

10. Find the inverse Laplace transform of $\dfrac{n!}{(s-a)^{n+1}}$ where $s > a$.

11. Let $u_c(t)$ be unit step function and $L(f(t)) = \mathrm{F}(s)$. Show that :

    $\mathrm{L}(u_c(t)f(t-c)) = e^{cs}\mathrm{F}(s)$.

12. Find the inverse Laplace transform of the following function by using the convolution theorem $\dfrac{1}{s^4(s^2+1)}$.

13. Solve the boundary value problem :

    $y'' + y = 0,\ y(0) = 0,\ y(\pi) = 0.$

14. Define an even function and show that if $f(x)$ is an even function then :

    $\int_{-\mathrm{L}}^{\mathrm{L}} f(x)\,dx = 2\int_{0}^{\mathrm{L}} f(x)\,dx.$

15. Define the following partial differential equations :
    (a) heat conduction equation.
    (b) one-dimensional wave equation.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Let $y_1(t)$ be a solution of $y' + p(t)y = 0$ and let $y_2(t)$ be a solution of $y' + p(t)y = g(t)$.

    Show that $y(t) = y_1(t) + y_2(t)$ is also a solution of equation $y' + p(t)y = g(t)$.

17. Find the value of b for which the following equation is exact, and then solve it using that value of b.

    $(xy^2 + bx^2y) + (x+y)x^2y' = 0.$

18. Solve the initial value problem

    $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2.$

19. Use method of variation of parameters find the general solution of :

$$y'' + 4y = 8\tan t, -\pi/2 < t < \pi/2.$$

20. Find the solution of the initial value problem :

$$2y'' + y' + 2y = \delta(t-5), y(0) = 0, y'(0) = 0.$$

here $\delta(t)$ denote the unit impulse function.

21. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0,\ y(0) = 3,\ y'(0) = -1.$$

22. Find the co-efficients in the Fourier series for f :

$$f(x) = \begin{cases} 0, & -3 < x < -1 \\ 1, & -1 < x < 1 \\ 0, & 1 < x < 3 \end{cases}$$

Also suppose that $f(x + 6) = f(x)$.

23. Find the solution of the following heat conduction problem :

$$100u_{xx} = u_t, 0 < x < 1, t > 0$$
$$u(0,t) = 0, u(1,t) = 0, t > 0$$
$$u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Find the general solution of the following differential equation using the method of integrating factors :

$$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0,1).

25. Find a series solution of the differential equation :

$$y'' + y = 0, -\infty < x < \infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau \ d\tau$

27. Find the temperature $u(x, t)$ at any time in a metal rod 50 cm long, insulated on the sides, which initially has a uniform temperature of 20°C throughout and whose ends are maintained at 0°C for all $t > 0$.

(2 × 10 = 20 marks)

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